

Mathematical Calculation Procedures and Drivers in Action in the Learning Environment

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Abstract

The paper reports on the qualitative results of the experimental phase of a study to examine the links between children's learning experiences associated with two digit division and the transactional analysis concept of drivers. The author presents results obtained from a process that used a questionnaire developed during a prior heuristic phase of research, combined with undergraduate student observations of the children, drawings produced by the children, and teacher observations on permission transactions used. Examples are provided for each of the five drivers.

Keywords

Mathematical learning environment; procedures for calculation of two digit division; drivers; permission transactions

Study Objectives

The aims of the research were to:

- investigate how different mathematical calculation procedures for two digit division, with increasing levels of difficulty, might activate drivers (Kahler 1975) with different levels of intensity;
- to explore the impact on the children's learning results of permission transactions (1966) used by teachers.

The Research Context

This paper reports on one part of a research process that has been developed over a period of about 7 years and will conclude in 2010. The heuristic phase ran from 2003 – 2005 and involved development of a questionnaire, originating with one devised by Klein (1984), and including

pictures and a structured interview guide, plus a grid and associated training in direct observation of driver behaviour. In the heuristic phase, four 4th year Primary school classes (98 children) and two 3rd year Primary school classes (46 children) were involved.

The Experimental Phase, which is the focus for this paper, ran from 2005 to 2007. During this phase, experiments were carried out according to the following procedures:

1. Administering evaluation entry tests.
2. Administering driver questionnaires
3. Starting up the didactic interventions, which included 5 activities in sequence; these were courses in class and study of drivers in action by means of the observation grid for: division with successive subtraction; division with the repeated addition method; division with the traditional method; division with simplified traditional method; division with traditional method.
4. Administration of final examinations and production by each child of a drawing of themselves during the test.
5. Organisation of recordings and observations in order to identify suitable descriptors and relations between the data.

In the experimental phase, four 4th year primary school classes were involved with a total of 93 children.

The Diffusion phase is now running from 2008 to 2010 and relates to the presentation of the first qualitative results and diffusion of the research.

The theoretical background

We may consider predominantly the structural characteristics of mathematics learning such as the processes of

abstraction, generalisation, transfer (Gagné R. 1985), and the method of representation using specific codes defining a language that requires an evolved and complex formalisation process. (Piu A., Fregola C., 2010).

As the concepts, rules, constructs and structures can be organised through mathematical language, the cognitive and meta-cognitive processes which enter into the area of the mathematics didactic influence are based on the evolved ability of thought, either that of procedural or declarative knowledge (Gagné E, 1989), requiring mechanical and elaborative activities whether it be knowledge that requires an already mastered know-how but that has to be reconfigured using the capacity for understanding, intuition, analysis, synthesis, decision-making and problem-solving (Resnick, 1987).

The aspects relative to the emotional sphere are connected to the fear of mathematics and to the common conviction that mathematics is a subject for a chosen few who are gifted with remarkable intelligence. It may be possible to intervene on some emotional-relational experiences, cognitive and meta-cognitive, which are connected to the history of each child and prevent, facilitate or influence the learning process of mathematics and motivational aspects. Bloom (1979) starts with the supposition that every subject matter can be understood as aimed at a *quality of instruction* that considers both cognitive and affective-interactional variables. Bloom separates affective suppositions from cognitive ones and demonstrates, by means of transversal and longitudinal studies, that there is a component of variability in scholastic progress given by such suppositions.

Emotions, feelings and moods which often cause frustration, do not always steer behaviour towards planning how much effort to make in order to achieve learning objectives in mathematics. Rather than being a resource for motivation, fear, which should represent the emotion of defence from danger or threats from the environment and anger, which should represent the emotion of the solution to those dangers and threats, reinforce the most common convictions about oneself when learning mathematics, about mathematics itself, the maths teacher and the teaching – aspects that make three convictions evident:

- inadequacy and inability, referring to oneself;
- inaccessibility, referring to mathematics
- inadequacy or incompetence, referring to the teacher and her/his teaching

In the didactic relationship there is the risk of perpetuating a vicious circle amongst these three factors which grow more and more apart and then go on to influence the motivation for learning mathematics.

If one enters “fear of mathematics” into a search engine, there are about 2,500,000 pages where the two words are present. Other words are also used which define a classification of the intensity of the fear, as an emotion, such as – anxiety, terror, anguish, panic. At the other extreme one would expect to find the emotion of joy as a reaction to successfully learning but instead there is a kind of “syndrome of the man in disguise” (Novellino 2003) a syndrome that affects students who are skilled in maths and is manifested by making them feel like supermen, intelligent and alien. In literature the fear of mathematics has been the subject of numerous studies which, in particular, can be traced to negative, unsuccessful experiences. Tobias (1993) introduces the expression Math Anxiety, pointing to the fear of making a mistake as one of the most important factors reported by students, to the method of reinforce/punish by the teachers, partial results attained by students and, again, the myth that the ability to learn is a special gift that cannot be affected by the method of learning

From the point of view of social communications this consequently introduces the adaptation of the students, the teachers and the parents to a model of behaviour which can tend to make the children justify themselves and yield to their lack of ability. Anne Siety (2003), psychopathologist and specialist in psycho-pedagogy of mathematics, focuses on the emotional aspect of mathematics and its consequences on the individual. The block in mathematics is not always caused by failing to understand the subject or a problem, that can be resolved with a little explanation and work, but the entire world of mathematics remains relentlessly closed and all this can result in panic or something more serious which is based on fear. According to the author, the origin of this fear in pupils is caused by their perception of mathematics as something unpleasant, incomprehensible since anyone who does not have a good head for maths will never be able to do it.

According to the constructivism theory, convictions are the fruit of a continuous process of interpretation of the reality effected by children and they develop with the implicit purpose of giving a sense to experiences with mathematics. Op'T Eynde (2002), describes the convictions of students relating to mathematics as those subjective conceptions which are possessed, implicitly or explicitly, that students consider as real, that is that influence their mathematical understanding and the solution to problems. Schoenfeld (1983) adds the importance of the environment in the generation of convictions. McLeod (1992) examines a classification that is different to that of Schoenfeld because it contains convictions on teaching mathematics rather than on convictions on the task.

Di Martino P. (2007) and Underhill (1988), take into consideration the convictions about oneself in a social context, and that of teaching and learning mathematics. Lewis (1990) divides the categories according to the ways

in which they can originate. However this criteria presents some problems because the same conviction can be found in different categories if held by two different people and it is not clear who stipulates the origin of a conviction in that not everyone is aware of them.

It is clear that these convictions can influence motivation and behaviour because from these come the convictions on what doing well in maths means, and what behaviour is required in order to be successful. However almost all studies on convictions are based on the research by Bandura and his Social foundations of Thought and Actions (1986). The author maintains that the convictions about oneself condition a certain form of control of thought and action. Bandura also underlines the importance of what he calls outcome expectations, or what the subject thinks will be the consequences of actions. Usually with the expression theory of success convictions about success and failure in mathematics are involved. These convictions have been the subject of psychological studies converging in the attributive theory: "Attributions can be defined as perceptions which individuals have concerning the causes of events which happen to themselves (self-attribution) or to others (hetero-attribution)" De Beni R., Moè A., (2000). Studies carried out by Fennema (1985) pointed out that males attributed their success to their own ability and their failure to lack of effort. In a different way, subsequent studies carried out by Schoenfeld (1989) demonstrated that, irrespective of gender, students who had less consideration of their own ability in mathematics tended to attribute any success to luck, and any failure to their own ability/inability.

Attitudes are understood to be internal, or mental and have a (favourable/unfavourable) direction and varying intensity and are connected to a predisposition to act (Zan. 1996). According to the simplest explanation, attitude is the level of positive or negative emotion associated with a particular object, and hence attitude to mathematics is simply a positive/negative inclination towards it. (McLeod 1989, Haladyna e al., 1983). A more articulated explanation sees three components in an attitude – an emotional reaction, convictions about the subject and behaviour towards the subject. From this perspective attitude towards the subject of mathematics is defined in a more articulated way than simply emotions which one associates with mathematics (that however retain a positive/negative value,) to include convictions which are held and behaviour which is triggered. (Hart. 1989) Therefore, the behaviour of a subject depends on more than one factor, into which concepts relating to perception of oneself can be integrated, including self-effectiveness (Bandura, 1993, 1997) and self-esteem (Convington, 1998).

Hackett and Betz (1989) quote evidence that self-effectiveness in mathematics is a predictor for results and performance. Schunk and Lilly (1984) emphasise the influence of self-esteem on results and performance in mathematics; perception of their ineffectiveness causes

pupils to lower their motivation for, and effort in, mathematics. Norwich (1987) showed similar results.

Linking the above to transactional analysis concepts led us to focus on two in particular – drivers and permission transactions. For the benefit of readers who may be unfamiliar with transactional analysis terminology, Kahler (1975) described drivers as behaviours that last from a split second to no more than seven seconds and reinforce an existential position of "I'm OK if". He linked this to Berne's (1972 p 344) notion that our script is "driven" by repetition compulsion so that drivers are microscopic repeats of our overall life pattern. Crossman (1966) identified permissions as transactions that effect a change in the direction of the recipient's behaviour; Stein (1971) referred to a permission as an attempt to realign the recipient with their original script-free state; and Woollams & Brown (1978) used the term permission transaction to indicate a message that "It's OK to.." Brook (1996) reviews the concept and provides a categorisation of permissions into affective, behavioural, cognitive and physiological.

The results of the study

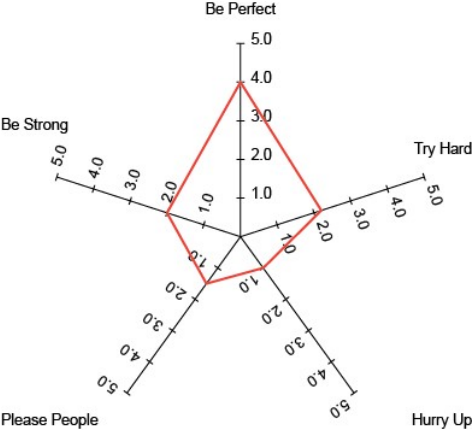

A report has been tabled for each child showing identification of the driver through the grid, the profile of the driver obtained by means of the questionnaire, the drawing of themselves, intervention of the teacher with the permission transactions, and results attained. In the following pages are 5 typical tables, one for each prevalent driver, representing one of the most important results of the research.

Verbal behaviour

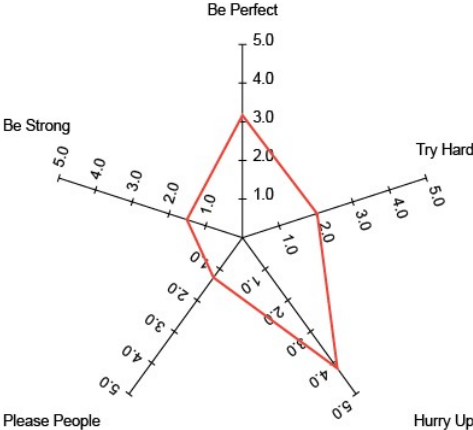

Some examples from the work carried out in the fourth year primary school; identification of drivers from analysis of verbal behaviour:

1. Mathematics is written in an exercise book with small squares and I like small squares because they help me to write well, neatly and precisely. Be Perfect Driver.
2. When I do division I can't find the number I need straight away and I start thinking that I am a bit crazy. Hurry Up Driver.
3. I try my best, but then everything seems so difficult, I have to try harder and then the numbers go missing and the sums never come out. Try Hard Driver.
4. I now just let things bounce off me, even if I can't do division I think it's pointless worrying about it because lots of my schoolmates can't do it either. Be Strong Driver
5. The teacher hasn't checked my maths exercise book, and I haven't done my homework because I only learned division for her. Please Others Driver.

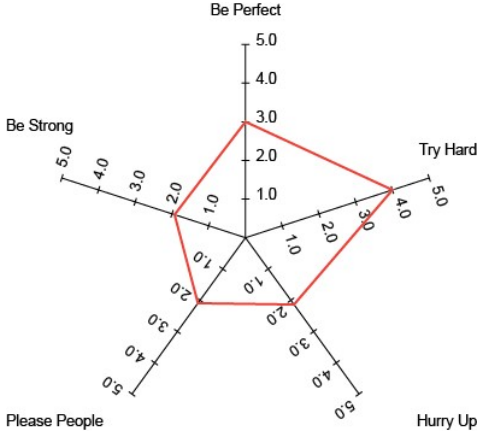

Child 1

Observations and grid	Questionnaire and Driver Profile	Drawing of myself	Behaviour before intervention of the teacher	Intervention of the teacher	Learning results
<p><i>Words:</i> “of course”, “I’ll do it again, I don’t like it like this”.</p> <p><i>Tones:</i> well modulated.</p> <p><i>Gestures:</i> brings the hand towards the chin.</p> <p><i>Positions:</i> upright and well balanced.</p> <p><i>Facial Expressions:</i> severe.</p> <p><i>Result:</i> Be perfect</p>	<p><i>Driver</i></p> 		<ul style="list-style-type: none"> ▪ Becomes agitated when a new activity is suggested. ▪ Concentrates on details and loses the thread. ▪ Conscientious 	<ul style="list-style-type: none"> ▪ Turned to the Normative Positive Parent. ▪ Transmitted confidence in pupil’s ability: emphasised the correct results achieved and at an appropriate time ▪ Praised the moments of concentration on the process rather than on the activity and unessential details. ▪ Gave permission to “be worth” something even though something is still missing ▪ Stimulated intuition. 	<ul style="list-style-type: none"> ▪ Attention time and functional concentration increased ▪ Accepts mistakes. ▪ Does not get lost in details. ▪ Accuracy means being able to deal better with the task ▪ More intuitive.

Child 2

Observations and grid	Questionnaire and Driver Profile	Drawing of myself	Behaviour before intervention of the teacher	Intervention of the teacher	Learning results
<p><i>Words:</i> “come on”.</p> <p><i>Tones:</i> up and down.</p> <p><i>Gestures:</i> brings the hand towards the chin.</p> <p><i>Positions:</i> continually fidgeting.</p> <p><i>Facial Expressions:</i> “inattentive”</p> <p><i>Result:</i> Hurry Up</p>	<p><i>Driver</i></p> 		<ul style="list-style-type: none"> ▪ Very competitive. ▪ Asks questions “for the sake of it” in the hope of getting useful information to arrive at the results. ▪ Is confident of never making a mistake. ▪ Skips steps ▪ Writes untidily 	<ul style="list-style-type: none"> ▪ Encouraged the pupil “to take his/her time”. ▪ Pointed out that at the end of the process a better result is obtained and in less time if “accuracy is displayed and the time necessary is made use of ▪ Reinforced attitudes to attention, accuracy in writing down the steps and operations. 	<ul style="list-style-type: none"> ▪ Compares self with others on specific subjects. ▪ Distinguishes the mistake from the error. ▪ Admits difficulties and has learnt to ask pertinent questions.

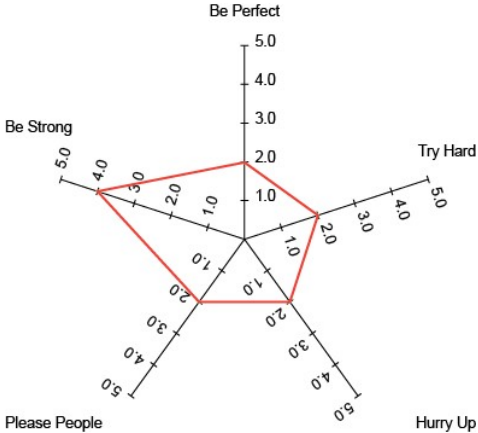

Child 3

Observations and grid	Questionnaire and Driver Profile	Drawing of myself	Behaviour before intervention of the teacher	Intervention of the teacher	Learning results
<p><i>Words:</i> "it's difficult", "I can't do it", "I'm not capable"</p> <p><i>Tones:</i> muted.</p> <p><i>Gestures:</i> moves tightened fists</p> <p><i>Positions:</i> bends forward, placing hands on knees.</p> <p><i>Facial Expressions:</i> frowns and screws up eyes</p> <p><i>Result:</i> Try Hard</p>	<p><i>Driver</i></p> 		<ul style="list-style-type: none"> ■ Becomes agitated when a test is suggested. ■ Freezes up when a mistake is made. ■ Concentrates on the effort and not on the activity ■ Asks for confirmation and Reassurance. ■ Resists and interrupts the test. 	<ul style="list-style-type: none"> ■ Used many positive strokes conditioned by event of activation and positive results ■ Transmitted confidence in pupil's ability: encouraged the commitment/result ■ Relation ■ Encouraged concentrating on the activity instead of saying "I can't do it". 	<ul style="list-style-type: none"> ■ Stays calm when activities are suggested. ■ Accepts mistakes and carries on. ■ Concentrates on the task. ■ Asks for help.

Child 4

Observations and grid	Questionnaire and Driver Profile	Drawing of myself	Behaviour before intervention of the teacher	Intervention of the teacher	Learning results
<p>Words: “come on”, “...will you help me?”, “is that right”</p> <p>Tones: alternates between high and low.</p> <p>Gestures: taps fingers and fidgets with legs.</p> <p>Positions: moves continually.</p> <p>Facial Expressions: sullen, tries to catch teacher’s eye.</p> <p>Result: Please Others</p>	<p><i>Driver</i></p>		<ul style="list-style-type: none"> ■ Keeps on asking for confirmation of correctness of operations ■ Easily changes method of attention 	<ul style="list-style-type: none"> ■ Encouraged to think about a single operation. ■ Pointed out the difficulties and mistakes of the child and discussed them with the child to make up for the prerequisites. ■ Encouraged the child to express <i>real</i> feelings. 	<ul style="list-style-type: none"> ■ Accepts and overcomes difficulties by discussing and asking for confirmation on the strategies adopted and no longer on the “correctness of what s/he has done” ■ Expresses feelings without expecting the teacher to confirm them. ■ At times is more connected to carrying out the task than getting approval of adult ■ Faces criticism without getting offended. ■ Depends less on opinion of others.

Child 5

Observations and grid	Questionnaire and Driver Profile	Drawing of myself	Behaviour before intervention of the teacher	Intervention of the teacher	Learning results
<p><i>Words:</i> does not use feelings.</p> <p><i>Tones:</i> flat, monotonous.</p> <p><i>Gestures:</i> rigid.</p> <p><i>Positions:</i> rigid.</p> <p><i>Facial Expressions:</i> hard, cold..</p> <p><i>Result:</i> Be Strong</p>	<p><i>Driver</i></p> 		<ul style="list-style-type: none"> ■ Does not express feelings. ■ Does not ask for help. ■ Very competitive 	<ul style="list-style-type: none"> ■ Encouraged the pupil to ask the teacher for help ■ Made sure pupil cooperated with the other children, asking her/him to help anyone who was left behind. 	<ul style="list-style-type: none"> ■ Pupil opened up and expressed feelings (laughs and jokes), both towards classmates and the teachers. ■ When in difficulty asks for help. ■ Not so competitive.

Relationship between learning results and drivers

The Be Perfect driver is what caused the greater number to abandon or temporarily halt the test.

The children with the Be Strong driver - and this surprised many - on the whole achieved excellent results, that is to say when anxiety is controlled one is more ready for mathematics. Less positive results were obtained in cases where, even though they have doubts or queries they do not ask or do not want help when the teacher approaches them. The Hurry Up and Try Hard drivers were noticed to be more frequent during the standard procedures for two digit division and it was noticed that in ninety per cent of the cases with low percentages of failure, only one or both were present. Finally, regarding the Hurry Up and Try Hard drivers, there is not always a correspondence between observations in the field and results of the questionnaire and this made us reassess the role of the questionnaire whilst not abandoning it.

The children with the Please Others driver are those who asked for more explanations and assistance, even pointing out their own deficiencies (signs of wooden leg game and clumsy idiot game – it wasn't for want of trying).

Discussion

We are still working on these results because we have felt it necessary to carry out a qualitative analysis of the research to be done on a wider range of children and, above all, to train the teachers to handle the instruction course themselves.

On the basis of the observations, we have implemented a course which aims at tackling, along with the teachers, the meaning of the drivers from a pedagogical angle. And we have noticed that it will be necessary to deal with the teachers' drivers and their effect on didactic interaction. We have provided the teachers with explanations about drivers and we have directed their attention to the fact that some aspects of the mechanism which characterise drivers involve very significant emotional characteristics.

The work has permitted us to observe essential aspects that are relevant when creating environments for learning mathematics (and also other subjects), within which the transactional analysis competence can be a deliberate part of the didactic activity and, at the same time, to consider to what extent some of these competences can become an integral part of the basic training for Primary School Teachers.

We feel that the method of managing the didactic relationship, and intervening at the right moment in emotional experiences, opens new perspectives for mathematical didactics and for didactics in general.

The author is a Certified Transactional Analyst (Educational), Mathematics Didactics for Integration Professor in Degree Courses of Primary School Teaching at University of Aquila and Lecturer in Mathematics and Psycho-pedagogy Didactic Masters Mathematics Art Science and Reality at Science of Teaching faculty University of Roma Tre. He can be contacted at fregola@uniroma3.it; cfregola@mathetica.it.

The work described was part of a study carried out at the Cattedra di Didattica Generale in the Degree Courses in Science of Primary School Teaching at University of Roma Tre and at the Cattedra di Matematica e Psicopedagogia Didactics Master in Mathematics in Art, Science and Reality in the same Degree courses. The author acknowledges the involvement of Prof. Daniela Olmetti Peja, Prof. Fabio Bocci, students Lia Amedei, Silvia Romano, Eliana Bilei, Laura Chiani and Roberta Di Fabio, teacher Isa Rossi and training supervisors Luisa Colcerasa, Maria Mansi, Viviana Rossanese, and thanks them for their participation, their proficient involvement and their attitude which was guided by their curiosity.

The research protocols are available for those who wish to consult them or start similar experiments – please contact the author.

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